

### 3 GOVERNING EQUATIONS FOR WATER QUALITY PARAMETERS

#### 3.1 Introduction

The water quality relationships implemented in RMA-11 are derived from QUAL2E and are presented in the sections that follow. The reader is referred to the Brown and Barnwell (1987) for more complete details. QUAL2E has multiple options for a number of the water quality terms. Currently RMA-11 operates with single selections from these options. The selected options are listed below.

1. Algae growth rate option number 2 (limiting nutrient)
2. Algae light function option number 1 (half saturation)
3. DO atmospheric reaeration rate option number 2 (Churchill et al. 1962)<sup>1</sup>
4. For steady state, light averaging option number 1 (FL calculated from total light/daylight hours, then scaled by the ratio of daylight hours in a day).

#### 3.2 Temperature Dependence of Rate Coefficients

The temperature values computed in RMA-11 are used to adjust the rate coefficients in the source / sink terms. These coefficients are input at 20°C and then corrected to the actual temperature using a Streeter-Phelps formulation:

$$X_t = X_{20} \theta^{(T-20)}$$

where

- |            |   |  |
|------------|---|--|
| $X_t$      | = | the value of the coefficient at the local computed temperature |
| $X_{20}$   | = | the value of the coefficient at 20°C                           |
| $\theta^*$ | = | an empirical constant for each reaction coefficient.           |

#### 3.3 Chlorophyll $\underline{a}$ and Algae Growth and Settling Rates

For this model chlorophyll  $\underline{a}$  is assumed to be directly proportional the algal biomass of phytoplankton. So that:

$$\text{Chl } \underline{a} = \alpha_0 A$$

where

- |                             |   |  |
|-----------------------------|---|--|
| $\text{Chl } \underline{a}$ | = | chlorophyll $\underline{a}$ concentration, ( $\mu\text{g-Chl } \underline{a}/\text{L}$ ) |
| $A$                         | = | algal biomass concentration, ( $\text{mg/L}$ )   |
| $\alpha_0$                  | = | a conversion factor, ( $\mu\text{g-Chl } \underline{a}/\text{mg } A$ )                   |

The net growth rate term for algae  $G_a$  may be written as:

$$G_a = \mu A - \rho A$$

where

- |          |   |  |
|----------|---|--|
| $\mu$    | = | local specific growth rate of algae - temperature adjusted , (1/day) |
| $\rho^*$ | = | local respiration rate of algae - temperature adjusted , (1/day)     |

and the settling rate for algae  $V_a$  is given by:

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<sup>1</sup> Two additional options not in QUAL2E have been added for reaeration computation.

$$\sigma_1^* = \frac{V_a}{\sigma_1} = \text{settling rate of algae - temperature adjusted, (m/day).}^+$$

A single growth rate formula is currently implemented in RMA-11. The form used is the limited nutrient approach where the growth rate is limited by lowest growth rate factor of the nutrients. In this form:

$$\mu = \mu_{\max} (FL) \text{ Min } (FN, FP)$$

where

$$\begin{aligned} \mu_{\max}^* &= \text{maximum specific growth rate, - temperature adjusted (1/day)} \\ FL &= \text{algal growth rate limitation factor for light} \\ FN &= \text{algal growth rate limitation factor for nitrogen} \\ FP &= \text{algal growth rate limitation factor for phosphorous.} \end{aligned}$$

### 3.3.1 Light limitation

The limitation factor for light FL uses a Monod expression to relate light intensity to the limiting factor. That is:

$$FL = \frac{I_z}{K_L + I_z}$$

where:

$$\begin{aligned} I_z &= \text{light intensity at a depth } z \text{ below the surface, (kJ/m}^2\text{/s).} \\ K_L^* &= \text{a half saturation coefficient for light, (kJ/m}^2\text{/s).} \\ z &= \text{depth variable, (m).} \end{aligned}$$

For the case of depth averaged modelling, the light intensity must be averaged over the depth, assuming an exponential reduction of light intensity. The following form results:

$$FL = \frac{1}{\lambda d} \frac{K_L + I}{K_L + I e^{-\lambda d}}$$

where:

$$\begin{aligned} \lambda &= \text{light extinction coefficient, (1/m).} \\ I &= \text{light intensity at the surface, (kJ/m}^2\text{/s).} \end{aligned}$$

all other terms have been previously defined

Note that values for I and  $\lambda$  must be entered in the appropriate data input lines for RMA-11 if a full temperature simulation is not undertaken.

### 3.3.2 Light Intensity

When a full meteorologic heat budget is undertaken, surface light intensity is computed as follows:

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\* Indicates an input value to RMA-11

+ Note that for depth averaged cases the settling rate becomes part of the growth rate term and the growth rate term becomes

$$G_a = \mu A - \rho A - \frac{\sigma_1}{d}$$

where

$$d = \text{depth of flow, (m).}$$

(1) for transient simulations, as the instantaneous light intensity integrated over the time interval divided by the time interval to give an average light intensity, or  
 (2) for steady state simulations, as an average using a 24 hour summation of light intensity divided by the number of daylight hours. In this case the final growth rate is scaled by the proportion of the number of daylight hours in a 24 hour day.

### 3.3.3 Algal Self Shading

The light extinction coefficient is coupled to the algae density using the non-linear equation, see Riley equation in Bowie et al. (1985):

$$\lambda = \lambda_0 + \lambda_1 \alpha_0 A + \lambda_2 (\alpha_0 A)^{2/3}$$

$\lambda_0^*$  = non algal portion of the light extinction coefficient, (1/m).  
 $\lambda_1^*$  = linear algal self shading coefficient, (1/m. 1/( $\mu\text{g-Chl } \underline{a}/\text{L}$ )).  
 $\lambda_2^*$  = non linear algal self shading coefficient, (1/m. 1/( $\mu\text{g-Chl } \underline{a}/\text{L}$ )<sup>2/3</sup>).

all other terms have been previously defined

Suggested default values from the literature for  $\lambda_1$  and  $\lambda_2$  are:

$$\lambda_1 = 0.0088 \text{ (1/m. 1/(\mu\text{g-Chl } \underline{a}/\text{L}))}$$

$$\lambda_2 = 0.054 \text{ (1/m. 1/(\mu\text{g-Chl } \underline{a}/\text{L})^{2/3})}$$

### 3.3.4 Algal Nutrient Relationships

The algal growth limitation factors for nitrogen (FN) and for phosphorous (FP) are defined by the Monod expressions:

$$FN = \frac{Ne}{K_N + Ne}$$

and

$$FP = \frac{P_2}{K_P + P_2}$$

where

$Ne$  = the effective local concentration of available inorganic nitrogen, (mg-N/L)  
 $K_N^*$  = the Michaelis - Menton half saturation constant for nitrogen, (mg-N/L)  
 $P_2$  = the local concentration of dissolved phosphorous, (mg-P/L).  
 $K_P$  = the Michaelis - Menton half saturation constant for phosphorous, (mg-P/L).

Algae are assumed to use ammonia and/or nitrate as a source of inorganic nitrogen. The effective concentration of available nitrogen  $Ne$  is given by:

$$Ne = N_1 + N_3$$

where

$N_1$  = concentration of ammonia nitrogen, (mg-N/L)  
 $N_3$  = concentration of nitrate nitrogen, (mg-N/L)

### 3.3a Bed Algae Net Growth Rate

As with water column algae (see Section 3.3) chlorophyll  $\underline{a}$  is assumed to be directly proportional the bed algal biomass of phytoplankton. So that:

where

$$\text{Chl } \underline{a} = \alpha_0 A_B$$

Chl  $\underline{a}$  = chlorophyll  $\underline{a}$  concentration, ( $\mu\text{g-Chl } \underline{a}/\text{L}$ )  
 $A_B$  = bed algal biomass concentration, ( $\text{mg/L}$ )  
 $\alpha_0$  = a conversion factor, ( $\mu\text{g-Chl } \underline{a}/\text{mg } A$ )

The net growth rate term for bed algae  $G_a$  may be written as:

$$G_a = (\mu_B^* - \rho_B^* - E_B - k_g) A_B$$

where

(1/day)

$\mu_B^*$  = local specific growth rate of bed algae - temperature adjusted,  
 $\rho_B^*$  = local respiration rate of bed algae - temperature adjusted, (1/day)  
 $E_B$  = incremental erosion coefficient for bed algae (1/day)  
 $k_g$  = bed algae mortality rate due to grazing (1/day)

A single growth rate formula is currently implemented in RMA-11. The form used is the limited nutrient approach where the growth rate is limited by lowest growth rate factor of the nutrients. In this form:

$$\mu_B = \mu_{B_{\max}} (FL_B) \text{Min} (FN_B, FP_B)$$

where

$\mu_{B_{\max}}^*$  = maximum specific growth rate, - temperature adjusted (1/day)  
 $FL_B$  = algal growth rate limitation factor for light  
 $FN_B$  = algal growth rate limitation factor for nitrogen  
 $FP_B$  = algal growth rate limitation factor for phosphorous.

### 3.3a.1 Light limitation

The limitation factor for light  $FL_B$  uses a Monod expression to relate light intensity to the limiting factor. That is:

$$FL_B = \frac{I_B}{K_L + I_B}$$

where:

$I_B$  = light intensity at bed, ( $\text{kJ/m}^2/\text{s}$ ).  
 $\lambda_B I_d$   
 $\lambda_B$  = bed algae self-shading factor  
 $I_d$  = light intensity at bottom of water column, ( $\text{kJ/m}^2/\text{s}$ ).  
 $I e^{-\lambda d}$   
 $I$  = light intensity at the surface, ( $\text{kJ/m}^2/\text{s}$ ).  
 $\lambda$  = light extinction coefficient (see Section 3.3.3), (1/m).  
 $d$  = depth of water column (m).  
 $K_L$  = a half saturation coefficient for light, ( $\text{kJ/m}^2/\text{s}$ ).

Note that values for  $I$  and  $\lambda$  must be entered in the appropriate data input lines for RMA-11 if a full temperature simulation is not undertaken.

### 3.3a.2 Light Intensity ( $I$ )

When a full meteorologic heat budget is undertaken, surface light intensity is computed as follows:

- (1) for transient simulations, as the instantaneous light intensity integrated over the time interval divided by the time interval to give an average light intensity, or
- (2) for steady state simulations, as an average using a 24 hour summation of light intensity divided by the number of daylight hours. In this case, the final growth rate is scaled by the proportion of the number of daylight hours in a 24 hour day.

### 3.3a.3 Bed algae sloughing and mass erosion

Bed algae erosion is modeled like surface erosion of cohesive sediments. That is, there are two forms of erosion representing incremental loss and mass wasting. Above a critical level of shear stress, algae with tenuous attachment to the bed will be pulled from the bed and carried away in the water column. At bed shear stresses just above this critical value, this “sloughing” occurs bit by bit at a rate proportional to the algae mass. At higher levels of stress the bulk shear stress of the bed may be exceeded and the bed will exhibit complete erosion of all bed algae. That is, when the shear stress exceeds the critical shear stress for mass erosion, the bed fails completely and all bed algae are instantly suspended.

Thus a model for the bed must include data defining an erosion rate for incremental erosion and the critical shear strength of the bed algae for both incremental and mass erosion.

The erosion rate for incremental erosion of bed algae may be written as:

$$E_B = M_B \frac{(\tau_b - \tau_{ci})}{\tau_{ci}} \quad \text{when } \tau_b > \tau_{ci}$$

where

$$\begin{aligned} \tau_b &= \text{bed shear stress (see section 3.8.1), (N/m}^2\text{).} \\ \tau_{ci} &= \text{critical shear stress for incremental erosion, (N/m}^2\text{).} \\ M_B &= \text{erodibility constant for bed algae, (/s)} \end{aligned}$$

Note that the erodibility of bed algae may be a function of length and type of algae making up the bed community. Because these attributes typically change with time of year, the erodibility constant may need to change with season.

The erosion rate for mass erosion can be defined assuming that all bed algae are eroded over a time step. In the model, when the critical shear for mass erosion is exceeded, bed algae concentration is set to zero. The form may be written as:

$$E_m = \frac{\Delta m}{\Delta t} \quad \text{when } \tau_b > \tau_{cm}$$

where

$$\begin{aligned} \Delta m &= \text{Mass eroded per unit bed area (gm/m}^2\text{)} \\ \Delta t &= \text{time step, (s).} \\ \tau_{cm} &= \text{critical shear stress for mass erosion of bed algae, (N/m}^2\text{).} \end{aligned}$$

### 3.3a.4 Bed Algal Nutrient Relationships

The bed algal growth limitation factors for nitrogen ( $FN_B$ ) and for phosphorous ( $FP_B$ ) are defined by the Monod expressions:

$$FN_B = \frac{N_{BE}}{K_N + N_{BE}}$$

and

$$FP_B = \frac{P_{B2}}{K_P + P_{B2}}$$

where

- $N_{BE}$  = the effective local concentration of inorganic nitrogen available in the bed, (mg-N/L)
- $K_N^*$  = the Michaelis - Menton half saturation constant for nitrogen, (mg-N/L)
- $P_{B2}$  = the local concentration of dissolved phosphorous, (mg-P/L).
- $K_P$  = the Michaelis - Menton half saturation constant for phosphorous, (mg-P/L).

Algae are assumed to use ammonia and/or nitrate as a source of inorganic nitrogen. The effective concentration of available nitrogen  $N_{BE}$  is given by:

$$N_{BE} = N_{B1} + N_{B3}$$

where

- $N_{B1}$  = concentration of ammonia nitrogen, (mg-N/L)
- $N_{B3}$  = concentration of nitrate nitrogen, (mg-N/L)

### 3.4 Nitrogen Cycle

The nitrogen cycle is assumed to consist of four nitrogen species that transform cyclically from organic nitrogen to ammonia to nitrite to nitrate.

#### 3.4.1 Organic Nitrogen

Organic nitrogen is created by respiration of algae and is lost by settling to the bottom and hydrolysis to ammonia. The equations for growth  $G_{N4}$  and settling  $V_{N4}$  of organic nitrogen are thus given by

$$\begin{aligned} G_{N4} &= \alpha_1 \rho A - \beta_3 N_4 \\ V_{N4} &= \sigma_4 \end{aligned}$$

where:

- $N_4$  = concentration of organic nitrogen, (mg-N/L)
- $\alpha_1$  = fraction of algal biomass that is nitrogen, (mg-N /mg-A).
- $\beta_3$  = rate constant for hydrolysis of organic nitrogen to ammonia nitrogen - temperature adjusted, (1/day).
- $\sigma_4$  = rate coefficient for settling of organic nitrogen usually expressed in m/day units - temperature adjusted, (m/day). Note that in QUAL2E this rate is expressed in 1/day units.

#### 3.4.2 Ammonia Nitrogen

Ammonia nitrogen is created by hydrolysis of organic nitrogen and by releases from benthic sources at the bottom. It is lost by biological oxidation and algal grazing. The choice of algal grazing on ammonia and nitrate nitrogen sources is dependent on a preference factor.

The equation for growth  $G_{N1}$  of ammonia nitrogen is thus given by:

$$G_{N1} = \beta_3 N_4 - \beta_1^* N_1 + \sigma_3/d^{++} - F_1 \alpha_1 \mu A$$

where

- $N_1$  = the concentration of ammonia nitrogen, (mg-N/L)
- $\beta_1^*$  = rate constant of biological oxidation of ammonia nitrogen - temperature adjusted, (1/day).
- $\sigma_3^*$  = the benthos source rate of ammonia nitrogen - temperature adjusted, (mg-N/m<sup>2</sup>-day)
- $F_1$  = fraction of algal nitrogen uptake from ammonia pool,

given by:

$$F_1 = P_N N_1 / [P_N N_1 + (1 - P_N) N_3]$$

where:

- $N_3$  = the concentration of nitrate nitrogen, (mg-N/L).
- $P_N$  = preference factor for ammonia nitrogen, ( 0.0 - 1.0 ) following Bowie et al. (1985). this equivalent to the fraction of nitrogen uptake from the ammonia pool when the concentrations of ammonia and nitrate nitrogen are equal.

all other terms have been previously defined

### 3.4.3 Nitrite Nitrogen

Nitrite nitrogen is created by oxidation from ammonia nitrogen and lost by oxidation to nitrate nitrogen..

The equation for growth  $G_{N2}$  of nitrite nitrogen is thus given by:

$$G_{N2} = \beta_1 N_1 - \beta_2 N_2$$

where:

- $N_2$  = the concentration of nitrite nitrogen, (mg-N/L)
- $\beta_2$  = rate constant of oxidation of nitrite nitrogen -temperature adjusted, (1/day).

all other terms have been previously defined

### 3.4.4 Nitrate Nitrogen

Nitrate nitrogen is created by oxidation from nitrite nitrogen and lost by uptake due to algal grazing.

The equation for growth  $G_{N3}$  of nitrate nitrogen is thus given by:

$$G_{N3} = \beta_2 N_2 - [1 - F_1] \alpha_1 \mu A$$

where all terms have been previously defined.

### 3.4.5 Inhibition of Nitrification at low Dissolve Oxygen

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<sup>++</sup>  $\sigma_3/d$  only appears as a source term in the equations for depth averaged elements, otherwise  $\sigma_3$  appears as a source at the lower boundary

RMA-11 follows QUAL2E by inhibiting the rate of nitrification at low levels of dissolved oxygen. This inhibition effect has been reported in DSIR (1964) and TWDB (1984).

Nitrification rates are modified by computing a factor (0 - 1.0) and then applying this factor to the nitrification rate coefficients for ammonia and nitrite conversion. it is computed using the following equation:

$$C0 = 1.0 - \exp(-KNITR * O)$$

where

C0	=	nitrification rate correction factor
KNITR	=	first order nitrification inhibition coefficient, (L/mg)
O	=	concentration of dissolved oxygen, (mg/L).

Values in the range 0.6 - 0.7 reportedly match reported data in the references above.

### 3.4a Bed Nitrogen

Be nitrogen is assumed to consist of two nitrogen species that transform cyclically from organic nitrogen to ammonia and released to the water column at times of low DO.

#### 3.4a.1 Organic Nitrogen

Organic nitrogen is created by the bottom and and lost by hydrolysis to ammonia. The equations for growth  $G_{Nb4}$  is thus given by

$$G_{Nb4} = \sigma_4 N_4 - \beta_{3b} N_{b4}$$

where:

$N_{b4}$	=	bed concentration of organic nitrogen, (g-N/m <sup>2</sup> )
$\beta_{3b}$	=	rate constant for hydrolysis of organic nitrogen to ammonia nitrogen - temperature adjusted, (1/day).
$\sigma_4$	=	rate coefficient for settling of organic nitrogen usually expressed in m/day units - temperature adjusted, (m/day). Note that in QUAL2E this rate is expressed in 1/day units.

#### 3.4a.2 Bed Ammonia Nitrogen

Ammonia nitrogen is created by hydrolysis of bed organic nitrogen. It is lost by to the water column at low DO in the water column.

The equation for growth  $G_{Nb1}$  of bed ammonia nitrogen is thus given by:

$$G_{Nb1} = \beta_{3b} N_{b4} - \sigma_{3b} N_{b1}$$

where

$N_{b1}$	=	the concentration of bed ammonia nitrogen, (g-N/m <sup>2</sup> )
$\sigma_{3b}$	=	loss rate to the water column when DO is below an input threshold. Thus $\sigma_{3b} = 0$ when DO is above threshold.

all other terms have been previously defined



### 3.5 Phosphorous Cycle

The phosphorous cycle is assumed to consist of two phosphorous species that are transformed cyclically from organic phosphorous to dissolved phosphorous. The cycle is completed by algal uptake and respiration.

#### 3.5.1 Organic Phosphorous

Organic phosphorous is created by respiration of algae and depleted by transformation to the dissolved form and by settling to the bottom.

The equations for growth  $G_{P1}$  and settling  $V_{P1}$  of organic phosphorous are thus given by

$$\begin{aligned} G_{P1} &= \alpha_2 \rho A - \beta_4 P_1 \\ V_{P1} &= \sigma_5 \end{aligned}$$

where:

$$\begin{aligned} P_{1*} &= \text{concentration of organic phosphorous, (mg-P/L)} \\ \alpha_{2*} &= \text{fraction of algal biomass that is phosphorous, (mg-P /mg-A).} \\ \beta_4 &= \text{rate constant for transformation of organic phosphorous to} \\ &\quad \text{dissolved phosphorous - temperature adjusted, (1/day).} \\ \sigma_5^* &= \text{rate coefficient for settling of organic phosphorous usually} \\ &\quad \text{expressed in m/day units - temperature adjusted, (m/day). Note} \\ &\quad \text{that in QUAL2E this rate is expressed in 1/day units.} \end{aligned}$$

All other terms have been previously defined.

#### 3.5.2 Dissolved Phosphorous

Dissolved phosphorous is created by transformation from the organic form and from benthic bottom sources. It is depleted by uptake as a nutrient by algae and by attachment to suspended sediment.

The equation for growth  $G_{P2}$  of dissolved phosphorous is thus given by:

$$G_{P2} = \beta_4 P_1 + \sigma_2 / d^{++} - \alpha_2 \mu A - \beta_5 P_2$$

where

$$\begin{aligned} P_{2*} &= \text{the concentration of dissolved phosphorous, (mg-P/L)} \\ \sigma_2^* &= \text{the benthos source rate of dissolved phosphorous - temperature} \\ &\quad \text{adjusted, (mg-P/m}^2\text{-day)} \\ \beta_5 &= \text{rate constant for attachment of dissolved phosphorous to} \\ &\quad \text{suspended sediments, (1/day).} \end{aligned}$$

All other terms have been previously defined.

#### 3.5a Bed Phosphorous Cycle

The phosphorous cycle is assumed to consist of two phosphorous species that are transformed cyclically from organic phosphorous to dissolved phosphorous. The cycle is completed by release to the water column at low DO.

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<sup>++</sup>  $\sigma_2 / d$  only appears as a source term in the equations for depth averaged elements, otherwise  $\sigma_2$  appears as a source at the lower boundary

### 3.5a.1 Bed Organic Phosphorous

Bed Organic phosphorous is created by settling from the water column and depleted by transformation to the dissolved form. The equation for growth  $G_{Pb1}$  is thus given by

$$G_{Pb1} = \sigma_5 P_1 - \beta_{4b} P_{b1}$$

where:

- $P_{b1}$  = bed concentration of organic nitrogen, (g-P/m<sup>2</sup>)
- $\beta_{4b}$  = rate constant for conversion of organic phosphorous to dissolved phosphorous - temperature adjusted, (1/day).
- $\sigma_5$  = rate coefficient for settling of organic phosphorous usually expressed in m/day units - temperature adjusted, (m/day). Note that in QUAL2E this rate is expressed in 1/day units.

### 3.5a.2 Bed Dissolved Phosphorous

Bed Dissolved phosphorous is created by transformation of bed organic phosphorous. It is lost by to the water column at low DO in the water column.

The equation for growth  $G_{Pb2}$  of bed dissolved phosphorous is thus given by:

$$G_{Pb2} = \beta_{4b} P_{b1} - \sigma_{5b} P_{b2}$$

where

- $P_{b2}$  = the concentration of bed dissolved phosphorous, (g-P/m<sup>2</sup>)
- $\sigma_{5b}$  = loss rate to the water column when DO is below an input threshold.  
Thus  
 $\sigma_{5b} = 0$  when DO is above threshold.

all other terms have been previously defined

## 3.6 Oxygen Cycle

The oxygen cycle is assumed to consist of dissolved oxygen and carbonaceous BOD, the other nutrient cycles also demand oxygen. The cycle is completed by reaeration and algal generation during growth and depletion during respiration.

### 3.6.1 Carbonaceous Biochemical Oxygen Demand

This parameter expresses potential carbonaceous demand for oxygen in the system. This demand is influenced by settling and flocculation of the particles exerting the demand which serve to reduce the effective demand without uptake of oxygen.

The equations for growth  $G_L$  and settling  $V_L$  of BOD are thus given by

$$\begin{aligned} G_L &= -K_1 L \\ V_L &= K_3 \end{aligned}$$

where

- $L$  = the concentration of ultimate carbonaceous BOD, (mg/L)
- $K_1$  = deoxygenation rate coefficient - temperature adjusted, (1/day).
- $K_3$  = rate coefficient for settling of - temperature adjusted, (m/day).  
Note that in QUAL2E this rate is expressed in 1/day units. This

term also incorporates the loss rate due to flocculation converted to an effective settling rate, see Thomas (1948)

### 3.6.2 Dissolved Oxygen

Dissolved oxygen is subjected to a complex series of interactions with many of the constituents in the model. Many of the rate terms have been defined earlier but to clarify the growth equation they will be relisted.

The sources of dissolved oxygen are:

- (a) reaeration, and
- (b) oxygen from photosynthesis during algal growth.

Oxygen sinks are:

- (a) oxygen uptake due to algal respiration,
- (b) oxidation of ammonia nitrogen to nitrite nitrogen,
- (c) oxidation of nitrite nitrogen to nitrate nitrogen,
- (d) carbonaceous BOD decay, and
- (3) sediment oxygen demand.

The equation for growth  $G_O$  is thus given by:

$$G_O = K_2 (O_s - O) + (\alpha_3 \mu - \alpha_4 \rho) A_1 - K_1 L - \alpha_5 \beta_1 N_1 - \alpha_6 \beta_2 N_2 - K_4 / d^{++}$$

where:

$O$	=	concentration of dissolved oxygen, (mg/L).
$O_s$	=	saturation concentration of dissolved oxygen at the local temperature and pressure, (mg/L)
$K_2^*$	=	reaeration rate - temperature adjusted, (1/day).
$\alpha_3$	=	the rate of oxygen production per unit of algal photosynthesis, (mg-O/mg-A).
$\alpha_4^*$	=	the rate of oxygen uptake per unit of algae respired, (mg-O/mg-A).
$\alpha_5^*$	=	the rate of oxygen uptake per unit of ammonia nitrogen oxidation, (mg-O/mg-N).
$\alpha_6^*$	=	the rate of oxygen uptake per unit of nitrite nitrogen oxidation, (mg-O/mg-N).
$\mu$	=	local specific growth rate of algae - temperature adjusted, (1/day)
$\rho^*$	=	local respiration rate of algae - temperature adjusted, (1/day)
$A$	=	algal biomass concentration, (mg/L)
$L$	=	the concentration of ultimate carbonaceous BOD, (mg/L)
$d$	=	water depth, (m)
$K_1^*$	=	deoxygenation rate coefficient - temperature adjusted, (1/day).
$K_4$	=	sediment oxygen demand rate - temperature adjusted, (mg/m <sup>2</sup> /day).
$\beta_1^*$	=	rate constant of biological oxidation of ammonia nitrogen - temperature adjusted, (1/day).
$\beta_2^*$	=	rate constant of oxidation of nitrite nitrogen - temperature adjusted, (1/day).
$N_1$	=	the concentration of ammonia nitrogen, (mg-N/L)

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<sup>++</sup>  $K_4 / d$  only appears as a sink term in the equations for depth averaged elements, otherwise  $K_4$  appears as a sink at the lower boundary

$N_2$  = the concentration of nitrite nitrogen, (mg-N/L)

### 3.6.3 Dissolved Oxygen Saturation Concentration

Solubility of dissolved oxygen is dependent on temperature, atmospheric pressure and concentrations of dissolved solids, RMA-11 limits the dependence to temperature and atmospheric pressure when temperature is a defined constituent.

The relationships take the following forms:

$$\log_e O_{sn} = -139.34410 + 1.575701E+05/T - 6.642308E+7/T_k^2 + 1.243800E+10/T_k^3 - 8.621949E+11/T_k^4 - (1./1.80655)*S_{al}*(3.1929E-2 - (1.9428E+1/T_k) + (3.8673E+3/T_k^2))$$

where

$O_{sn}$  = equilibrium oxygen concentration at 1.00 atm, (mg/L).  
 $T_k$  = water temperature in ° K , within the range 0.0 to 40.0 ° C  
 $S_{al}$  = salinity (if active in the simulation).

and

$$O_s = O_{sn} P^{\frac{\{1 - P_{wv}/P\}\{1 - \phi P\}}{\{1 - P_{wv}\}\{1 - \phi\}}}$$

where  $O_s$  = equilibrium oxygen concentration at non standard pressure, (mg/L).  
 $P$  = atmospheric pressure, (atm).  
 $P_{wv}$  = partial pressure of water vapour, (atm), which may be computed from:

$$\log_e P_{wv} = 11.8571 - 3840.70/T_k - 216961/T_k$$

and

$\phi$  =  $0.000975 - 1.426E-5 T + 6.436E-8 T^2$   
 $T$  = temperature in ° C.

### 3.6.4 Atmospheric Reaeration Coefficient

QUAL2E offers the user a number of options for computation of reaeration coefficients. These are generally functions of depth and stream velocity. RMA-11 currently allows three options:

1. An adaptation of the formula given by Churchill et. al. (1962),
2. A formula based on wind speed suitable for ocean environments given by Liss and Merlivat (1986),
3. A constant rate input by the user.

The Churchill method (option 0) has been modified to express that rate as a first order source at the surface for other than depth averaged simulations. Thus, the original form is multiplied by depth. The rate equation then takes the following form:

$$K_2^{20} = 5.026 u_s^{0.969} d^{-0.673}$$

where

$K_2^{20}$  = reaeration coefficient at 20 ° C, (m/day).  
 $d$  = average stream depth, (m).  
 $u_s$  = water velocity at the surface, (m/sec).

The wind dependent formula (option 1) for ocean environments is given by:

$$K_2^{20} = 0.048 u_w \quad u_w < 3.6 \text{ m/sec}$$

$$K_2^{20} = 0.773 u_w - 2.61 \quad 3.6 < u_w < 13.0 \text{ m/sec}$$

$$K_2^{20} = 1.600 \cdot u_w - 13.3 \quad 13.0 < u_w$$

where

$$u_w = \text{wind speed in m/sec.}$$

The constant formula (option 2) uses the input minimum rate as a constant for all conditions.

At low water or wind speeds, options 0 and 1 are both subject to a minimum value that is part of the input data.

### 3.7 Arbitrary Non-Conservative Constituents

RMA-11 permits up to 6 arbitrary non-conservative constituents. Each of these constituents may decay, settle or have a benthic source from the bottom. Additionally these constituents may be coupled together in a first order mode so that any constituent may be a source or sink for the other. The equations below list the most general structure that may be input into the model.

The equations for growth  $G_{Ri}$ , settling  $V_{Ri}$  and benthic sources  $B_{Ri}$  of an arbitrary constituent are then given by

$G_{Ri} = -\sum K_{6ij} R_j$  where the summation extends over each of the defined arbitrary constituents and may include  $i$  itself, and

$$\begin{aligned} V_{Ri} &= \sigma_{6i} \\ B_{Ri} &= \sigma_{7i} \end{aligned}$$

$$\begin{aligned} R_j &= \text{concentration of the non-conservative constituent } j, \text{ (mg/L).} \\ K_{6ij} &= \text{decay rate or uptake rate between arbitrary constituents } i \text{ and } j - \\ &\quad \text{temperature adjusted, (1/day).} \\ \sigma_{6i} &= \text{settling rate for arbitrary constituent } i - \text{temperature adjusted,} \\ &\quad \text{(m/day)} \\ \sigma_{7i} &= \text{benthos source rate for arbitrary constituent } i - \text{temperature} \\ &\quad \text{adjusted, (mg/m}^2\text{-day)} \end{aligned}$$

If the constituent is coupled to itself then the  $K$  factor is a decay such that  $C=C_0 \cdot e^{-kT}$  Negative  $K$  for decay

If the constituent is linked to a different constituent, then  $C=C_0 + k_i \cdot C_i \cdot \Delta t$  where  $k$  is 1/day and  $\Delta t$  is in days.

Note that for the second case there is nothing to stop a constituent going negative

### 3.8 Suspended Sediment (Cohesive)

RMA-11 is constructed to permit transport of fine cohesive sediment or mud as an option of the model. several processes are modelled. Deposition through settling, erosion either as surface process or a mass process, and the development history of the bed layer. Erosion and/or deposition are dependent on the bed shear stress developed by flowing water, and the shear strength of surface layer of the bed. The strength of lower layers of the bed are increase through consolidation. This process is approximately modelled. The methodology on which this is based is that of Ariathurai and Krone (1976).

### 3.8.1 Cohesive Sediment Bed Shear Stress

RMA-11 provides two options for computation of bed shear stress, one based on a law of the wall with the wall assumed smooth. In this case bed shear stress is given by

$$\tau_b = \rho_w u_*^2$$

where

$$u_* = \text{shear velocity, (m/sec).}$$

$$\rho_w = \text{water density, (kg/m}^3\text{).}$$

then

$$u_m = u_* \log_e \left( \frac{3.32 u_* d}{\nu} \right) / V_k$$

where:

$$u_m = \text{mean flow velocity, (m/sec).}$$

$$d = \text{water depth, (m).}$$

$$\nu = \text{kinematic viscosity of water, (m}^2\text{/sec).}$$

$$V_k = \text{Von Karman's Constant}$$

For the case of a rough bed:

$$u_* = \frac{V_k u_m}{\log_e \{12.27 d/r\}}$$

where

$$r = \text{roughness height in metres.}$$

### 3.8.2 Cohesive Sediment Settling Rate

In an effort to model effective flocculation and its influence on settling rates the particle settling rate is optionally defined as a function of the suspended sediment concentration. The governing equation for settling velocity  $V_s$  is thus:

$$\begin{aligned} V_s &= V_1 \text{ for concentration } S < S_1 \\ V_s &= V_{sk} S^{p_c} \text{ for } S_1 < S < S_2 \\ V_s &= V_{sk} S_2^{p_c} \text{ for } S_2 < S \end{aligned}$$

where:

$V_1$ ,  $S_1$ ,  $S_2$  and  $p_c$  are all input parameters and  $V_{sk}$  is derived as matching condition.

### 3.8.3 Cohesive Sediment Bed Settling

When the shear stress on the bed is not sufficient to re-suspend particles that contact and bond with the bed, deposition occurs. The likelihood for settling to occur increases as the shear stress decreases relative to the critical shear stress. Thus the rate of settling  $V_b$  at the bed may be written as:

$$V_b = V_s \frac{\{\tau_d - \tau_b\}}{\tau_d} \quad \text{when } \tau_b < \tau_d$$

where:

$V_b$	=	effective bed settling velocity, (m/sec)
$V_s$	=	particle settling velocity, (m/sec).
$\tau_b$	=	bed shear stress for settling, (N/m <sup>2</sup> ).
$\tau_d^*$	=	critical shear stress for settling, (N/m <sup>2</sup> ).

### 3.8.4 Cohesive Sediment Erosion

Above a critical level of shear stress erosion of the bed will occur. Resistance of a cohesive bed to erosion by flowing water depends on a number of factors:

- The type of clay minerals that constitute the bed.
- The structure of the bed, which in turn depends on the environment in which the aggregates that formed the bed were deposited, the elapsed time, the temperature and the rate of gel formation.
- The chemical composition of the pore and eroding fluids.
- The stress history, i. e., the maximum overburden pressure the bed has experienced and the elapsed time at various stress levels.
- The presence of organic matter and its state of oxidation.

At bed shear stresses just above the critical value, erosion occurs particle by particle. This process is called surface erosion. At higher levels of stress the bulk shear stress of the bed may be exceeded and a portion of the bed may be liable to exhibit mass erosion. That is, when the shear stress exceed the critical shear stress for that portion of the bed, it fails completely and is instantly suspended.

Thus a model for the bed must include data defining the critical shear strength of each stratum of the bed and an erosion rate for surface erosion.

The erosion rate for surface erosion may be written as:

$$E_s = M \frac{\{\tau_b - \tau_{ce}\}}{\tau_{ce}} \quad \text{when } \tau_b > \tau_{ce}$$

where

$\tau_{ce}^*$	=	critical shear stress for erosion, (N/m <sup>2</sup> ).
$M$	=	erodibility constant

The erosion rate for mass erosion can be defined assuming that all mass is eroded over a time step. The form may be written as:

$$E_m = \frac{\Delta m}{\Delta t} \quad \text{when } \tau_b > \tau_{cl}$$

where

$\Delta m$	=	Mass eroded per unit bed area (gm/m <sup>2</sup> )
$\Delta t$	=	time step, (s).
$\tau_l^*$	=	critical shear stress for mass erosion for layer l, (N/m <sup>2</sup> ).

## 3.9 Sand Transport

In RMA-11 sand transport is modelled using the concept of sand transport potential. A sand transport formulation is used to define this potential which is assumed to depend on sand and flow parameters. In general terms the bed source or sink term is given by

$$S = \frac{C_{eq} - C}{t_c}$$

where

S	=	Source term, (gm/s/m <sup>2</sup> /meter of depth)
C <sub>eq</sub>	=	Equilibrium concentration (transport potential), (mg/l).
C	=	Sand concentration in the water
t <sub>c</sub>	=	Characteristic time for effecting the transition, this depends on

whether deposition or erosion is occurring,(s).

It is the selection of t<sub>c</sub> that presents the most empirical step of the analysis. This model broadly follows the method used in STUDH (Corps of Engineers 1985).

Five options are allowed for computation of the transport potential:

1. The Ackers-White (1973) equations which are presented below.
2. The Van Rijn (1984a,b,c,d ) method derived from the 1984 papers:
3. The Brownlie (1981) method:
4. Van Rijn (1989) method that allows for wave transport effects
5. Van Rijn (1993) method that expands the method sed for wave transport effects

### 3.9.1 Ackers-White Equations<sup>2</sup>

$$D_{gr} = D_{50} g^{1/3} (S_{gs} - 1.0)^{1/3} \nu^{*-2/3}$$

where

D <sub>gr</sub>	=	Grain number
D <sub>50</sub>	=	50th percentile grain diameter, (mm).
g	=	Acceleration due to gravity in English units for Ackers White formulation, (ft/s <sup>2</sup> ).
S <sub>gs</sub>	=	Specific gravity of sand
ν*	=	Kinematic viscosity of water in English units for Ackers White formulation, (ft <sup>2</sup> /s).

For D<sub>gr</sub> > 60 then four coefficients are defined:

n	=	0
A	=	0.17
m	=	1.5
c	=	0.025

For 60 > D<sub>gr</sub> > 1 then:

n	=	1.0 - 0.56 log D <sub>gr</sub>
A	=	$\frac{0.23}{D_{gr}^{1/2}} + 0.14$
m	=	$\frac{9.66}{D_{gr}} + 1.34$
log c	=	2.86 log D <sub>gr</sub> - (log D <sub>gr</sub> ) <sup>2</sup> - 3.53

<sup>2</sup> Note that the units for this formula are English.



$$u^* = \frac{v^*}{32^{1/2} \log \frac{10 d^*}{D_{50}}}$$

where

$$\begin{aligned} v^* &= \text{water velocity in English units for Ackers White formulation, (ft/s)} \\ d^* &= \text{depth in English units for Ackers White formulation, (ft).} \end{aligned}$$

The mobility number  $F_{gr}$  is defined by

$$F_{gr} = \frac{u^{*n} u^{*(1-n)}}{(g D_{50} (S_{gs} - 1.0))^{0.5}}$$

where

$$u^* = \text{bed shear velocity in English units for Ackers White formulation, (ft/s)}$$

Then the equilibrium concentration (transport potential)  $C_{eq}$  is given by

$$C_{eq} = c S_{gs} \left( \frac{v}{u^*} \right)^n \left( \frac{F_{gr}}{A} - 1.0 \right)^m$$

### 3.9.2 Shear Velocity

In the current version of the model the bed shear velocity  $u^*$  is computed using a Manning shear stress equation. Thus  $u^*$  in metric units is given by

$$u^* = \frac{M_n g^{0.5} v}{d^{1/6}}$$

where

$$\begin{aligned} M_n &= \text{Manning coefficient} \\ v &= \text{Water velocity near the bed, (m/s)} \end{aligned}$$

### 3.9.3 Sand Deposition

When the equilibrium sand concentration is less than the sand concentration in the water in the expression above deposition will occur. The characteristic time is then a function of fall velocity  $R_{ma-11}$  now offers two options for computation of  $t_c$ .

Option 0 is based on STUDH., it treats the time step as a limiting value. It is formulated as follows

$$t_c = \begin{aligned} &\text{larger of} && C_d \frac{d}{V_s} \\ &\text{or} && \Delta t \end{aligned}$$

where

$$\begin{aligned} C_d &= \text{Coefficient for deposition, (typical value 1.0).} \\ V_s &= \text{Fall or settling velocity, (m/s).} \\ \Delta t &= \text{Computation time step, (s)} \end{aligned}$$

Option 1 is an alternate formulation. It uses an input characteristic time as a limiting value and is thus independent of  $\Delta t$ . It is formulated as follows

$$t_c = \begin{aligned} &\text{larger of} && \frac{d}{V_s} \\ &\text{or} && 3600 * C_{td} \end{aligned}$$

where

$C_{td}$  = An input characteristic time for deposition, (typical value 0.05 hrs) and the other terms are already defined.

### 3.9.4 Sand Erosion

When the equilibrium sand concentration is greater than the sand concentration in the water erosion occurs. The characteristic time is then a more complex function for which only empirical forms have been used.

Once again two options have been included in RMA-11

Option 0 follows the methodology of STUDH, it treats the time step as a limiting value. The expression used is:

$$t_c = \begin{matrix} \text{larger of} \\ \text{or} \end{matrix} \begin{matrix} C_e \frac{d}{v} \\ \Delta t \end{matrix}$$

where

$$\begin{matrix} C_e & = & \text{Coefficient for erosion, (typical value 10.0).} \\ v & = & \text{Water velocity near the bed, (m/s).} \end{matrix}$$

Option 1 is an alternate formulation. It uses an input characteristic time as a limiting value and is thus independent of  $\Delta t$ . It is formulated as follows

$$t_c = \begin{matrix} \text{larger of} \\ \text{or} \end{matrix} \begin{matrix} \frac{d}{v} \\ 3600 * C_{te} \end{matrix}$$

where

$C_{te}$  = An input characteristic time for erosion, (typical value 0.05 hrs) and the other terms are already defined.

### 3.9.5 Bed structure

RMA-11 assumes that the bed is given an initial thickness and that deposition builds a bed with a porosity of 0.4. Thus, the increase of thickness from deposition ( $\delta$  +ve) or loss through erosion ( $\delta$  -ve) is given by

$$\Delta b = \delta \Delta t / (0.4 \rho_s)$$

where

$$\begin{matrix} \Delta b & = & \text{Additional thickness of sand, (m).} \\ \delta & = & \text{Source rate of sand, (kg/s/m}^2\text{)} \\ \rho_s & = & \text{Density of sand (kg/m}^3\text{).} \end{matrix}$$

During erosion the bed thickness serves to reduce the source rate if the bed thickness would be reduced to less than zero.

## **3.10 Coliform**

In RMA-11, coliform transport is modelled using three loss parameters:

1. settling
2. decay in darkness
3. light sensitive decay.

The equation for growth  $G_C$  is thus given by:

$$G_C = - (K_{c1} + K_{c2} + K_{c3}/d^{++}) C_c$$

where

$$\begin{aligned} C_c &= \text{the concentration of coliform, (mpn/100mL)} \\ K_{c1} &= \text{coliform die off rate in darkness - temperature adjusted (1/hr)} \end{aligned}$$

Note that this is input to the model as  $T_{90n}$  in hours and thus:

$$K_{c1} = 2.3026 \frac{1}{T_{90n}}$$

$$\begin{aligned} K_{c2} &= \text{coliform die off rate due to light - temperature adjusted (1/hr)} \\ K_{c3} &= \text{coliform settling rate, - temperature adjusted (m/hr).} \end{aligned}$$

$K_{c2}$  may be calculated using one of two options:

For option (0)

$K_{c2}$  is dependent on the light intensity and is given for depth averaged cases by

$$K_{c2} = 2.3026 \frac{[L_i \frac{(1 - \exp(-\lambda d))}{(\lambda d)}]^{0.7}}{L_c}$$

and for three-dimensional cases by

$$K_{c2} = 2.3026 \frac{[L_i \exp(-\lambda z_d)]^{0.7}}{L_c}$$

where:

$$\begin{aligned} L_i &= \text{light intensity expressed in MJ/m}^2\text{/hr} \\ L_c &= \text{coliform light coefficient (hr.[MJ/m}^2\text{/hr]}^{0.7}) \\ \lambda &= \text{light extinction coefficient (1/m)} \\ z_d &= \text{depth below water surface for 3-D (m)} \\ d &= \text{water depth for 2-D (m)} \end{aligned}$$

For option(1)

$K_{c2}$  is dependent only on the presence of solar radiation (sun up) and for this condition is computed as:

$$K_{c2} = 2.3026 \frac{1}{L_{cd}}$$

where:

$$L_{cd} = 90\% \text{ decay time coliform coefficient for daylight in hours}$$

### 3.11 Definition Of Symbols And Range Of Values Used In Water Quality Equations

$$\begin{aligned} \alpha_0^* &= \text{Chl } a \text{ to algal biomass conversion factor, } (\mu\text{g-Chl } a/\text{mg A}) & (10-100) \\ \alpha_1^* &= \text{fraction of algal biomass that is nitrogen, (mg-N /mg-A).} & (0.07-0.09) \\ \alpha_2^* &= \text{fraction of algal biomass that is phosphorous, (mg-P /mg-A).} & (0.01-0.02) \end{aligned}$$

++  $K_4/d$  only appears as a sink term in the equations for depth averaged elements, otherwise  $K_4$  appears as a sink at the lower boundary

$\alpha_3^*$	=	the rate of oxygen production per unit of algal photosynthesis , (mg-O/mg-A). (1.4-1.8)
$\alpha_4^*$	=	the rate of oxygen uptake per unit of algae respired, (mg-O/mg-A). (1.6-2.3)
$\alpha_5^*$	=	the rate of oxygen uptake per unit of ammonia nitrogen oxidation, (mg-O/mg-N). (3.0-4.0)
$\alpha_6^*$	=	the rate of oxygen uptake per unit of nitrite nitrogen oxidation, (mg-O/mg-N). (1.00-1.14)
$\beta_1^*$	=	rate constant of biological oxidation of ammonia nitrogen - temperature adjusted, (1/day). (0.1-1.0)
$\beta_2^*$	=	rate constant of oxidation of nitrite nitrogen -temperature adjusted, (1/day). (0.2-1.0)
$\beta_3^*$	=	rate constant for hydrolysis of organic nitrogen to ammonia nitrogen - temperature adjusted, (1/day). (0.02-0.40)
$\beta_4^*$	=	rate constant for transformation of organic phosphorous to dissolved phosphorous - temperature adjusted, (1/day). (0.01-0.70)
$\beta_5^*$	=	rate constant for removal of phosphate phosphorous by adhesion of sediments, (1/day). (0.00-0.10)
$\delta$	=	source rate of sand, (kg/s/m <sup>2</sup> )
$\Delta b$	=	additional thickness of sand bed, (m).
$\Delta t$	=	Computation time step, (s)
$\Delta m$	=	Mass eroded per unit bed area (gm/m <sup>2</sup> )
$\lambda$	=	light extinction coefficient, (1/m).
$\lambda_B$	=	bed algae self-shading factor
$\lambda_0^*$	=	non algal portion of the light extinction coefficient, (1/m).(variable)
$\lambda_1^*$	=	linear algal self shading coefficient, (1/m. 1/( $\mu\text{g-Chl } a/L$ )). (0.006-0.060)
$\lambda_2^*$	=	non linear algal self shading coefficient, (1/m. 1/( $\mu\text{g-Chl } a/L$ ) <sup>2/3</sup> ). (0.50)
$\mu$	=	local specific growth rate of algae - temperature adjusted , (1/day)
$\mu_{\max}^*$ (1.0-3.0)	=	maximum specific growth rate, - temperature adjusted (1/day)
$\mu_B$	=	local specific growth rate of bed algae (1/day)
$\mu_{B\max}$	=	maximum specific growth rate (1/day)
$\nu$	=	kinematic viscosity of water, (m <sup>2</sup> /sec).
$\nu^*$	=	Kinematic viscosity of water in English units for Ackers White formulation, (ft <sup>2</sup> /s).
$\rho^*$	=	local respiration rate of algae - temperature adjusted , (1/day) (0.05-0.50)
$\rho_B$	=	local respiration rate of bed algae (1/day)
$\rho_s$	=	density of sand (kg/m <sup>3</sup> ).
$\rho_w$	=	water density, (kg/m <sup>3</sup> ).
$\sigma_1^*$	=	settling rate of algae - temperature adjusted , (m/day). (0.15-1.82)
$\sigma_2^*$	=	the benthos source rate of dissolved phosphorous - temperature adjusted, (mg-P/m <sup>2</sup> -day) (variable)
$\sigma_3^*$	=	the benthos source rate of ammonia nitrogen - temperature adjusted, (mg-N/m <sup>2</sup> -day) (variable)

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\* Indicates an input value to RMA-11

$\sigma_4^*$	=	rate coefficient for settling of organic nitrogen usually expressed in m/day units - temperature adjusted, (m/day). Note that in QUAL2E this rate is expressed in 1/day units.	(0.002-0.200)
$\sigma_5^*$	=	rate coefficient for settling of organic phosphorous usually expressed in m/day units - temperature adjusted, (m/day). Note that in QUAL2E this rate is expressed in 1/day units.	(0.002-0.200)
$\sigma_{6i}$	=	settling rate for arbitrary constituent i - temperature adjusted, (m/day)	(variable)
$\sigma_{7i}$	=	benthic source rate for arbitrary constituent i - temperature adjusted, (mg/m <sup>2</sup> -day)	(variable)
$\theta^*$	=	an empirical constant for each reaction coefficient.	
$\tau_l^*$	=	critical shear stress for mass erosion for layer l, (N/m <sup>2</sup> ).	
$\tau_b$	=	bed shear stress for settling, (N/m <sup>2</sup> ).	
$\tau_{ce}^*$	=	critical shear stress for erosion, (N/m <sup>2</sup> ).	
$\tau_{cm}$	=	critical shear stress for mass erosion of bed algae, (N/m <sup>2</sup> ).	
$\tau_{ci}$	=	critical shear stress for incremental erosion of bed algae, (N/m <sup>2</sup> ).	
$\tau_d^*$	=	critical shear stress for settling, (N/m <sup>2</sup> ).	
A	=	algal biomass concentration, (mg/L)	
Chl <u>a</u>	=	chlorophyll <u>a</u> concentration, (µg-Chl <u>a</u> /L)	
$C_c$	=	the concentration of coliform, (mpn/100mL)	
$C_d$	=	coefficient for deposition,	(typical value 1.0).
$C_e$	=	coefficient for erosion,	(typical value 10.0).
$C_{eq}$	=	equilibrium concentration (transport potential), (mg/L)	
d	=	depth of flow, (m)	
d*	=	depth in English units for Ackers White formulation, (ft).	
$D_{gr}$	=	Grain number	
$D_{50}$	=	50th percentile grain diameter, (mm).	
$E_B$	=	incremental erosion coefficient for bed algae (1/day)	
$F_1$	=	fraction of algal nitrogen uptake from ammonia pool,	
FL	=	algal growth rate limitation factor for light	
FN	=	algal growth rate limitation factor for nitrogen	
FP	=	algal growth rate limitation factor for phosphorous.	
g	=	Acceleration due to gravity in English units for Ackers White formulation, (ft/s <sup>2</sup> ).	
I	=	light intensity at the surface, (kJ/m <sup>2</sup> /sec).	
$I_z$	=	light intensity at a depth z below the surface, (kJ/m <sup>2</sup> /sec).	
$k_g$	=	bed algae mortality rate due to grazing (1/day)	
$K_1^*$	=	deoxygenation rate coefficient - temperature adjusted, (1/day).	(0.02-3.5)
$K_2$	=	reaeration rate - temperature adjusted, (1/day).	(0.0-100.)
$K_{c1}$	=	coliform die off rate in darkness - temperature adjusted (1/hr)	(1/7.0)
$K_{c2}$	=	coliform die off rate due to light - temperature adjusted (1/hr)	
$K_{c3}$	=	coliform settling rate, - temperature adjusted(m/hr).	
$K_2^{20}$	=	reaeration coefficient at 20 ° C, (m/day).	
$K_3^*$	=	rate coefficient for settling of BOD usually expressed in m/day units - temperature adjusted, (m/day). Note that in QUAL2E this rate is expressed in 1/day units. This term also incorporates the loss rate due to flocculation converted to an effective settling rate.	(0.00-0.75)
$K_{\delta ij}$	=	decay rate or uptake rate between arbitrary constituents i and j - temperature adjusted, (1/day).	(variable)

$K_L^*$ =	a half saturation coefficient for light, (kJ/m <sup>2</sup> /sec).	(0.0037-0.0185)
$K_N^*$ =	the Michaelis - Menton half saturation constant for nitrogen, (mg-N/L)	(0.01-0.30)
$K_P^*$ =	the Michaelis - Menton half saturation constant for phosphorous, (mg-P/L).	(0.001-0.050)
$L$ =	the concentration of ultimate carbonaceous BOD, (mg/L)	
$L_i$ =	light intensity expressed in MJ/m <sup>2</sup> /hr	
$L_c$ =	coliform light coefficient (hr.[MJ/m <sup>2</sup> /hr] <sup>0.7</sup> )	(3.3)
$M$ =	erodibility constant	
$M_B$ =	erodibility constant for bed algae, (/s)	
$M_n$ =	Manning coefficient	
$N_1$ =	concentration of ammonia nitrogen, (mg-N/L)	
$N_2$ =	the concentration of nitrite nitrogen, (mg-N/L)	
$N_3$ =	concentration of nitrate nitrogen, (mg-N/L)	
$N_4$ =	concentration of organic nitrogen, (mg-N/L)	
$N_e$ =	the effective local concentration of available inorganic nitrogen, (mg-N/L)	
$O$ =	concentration of dissolved oxygen, (mg/L).	
$O_s$ =	saturation concentration of dissolved oxygen at the local temperature and pressure, (mg/L)	
$O_{sn}$ =	equilibrium oxygen concentration at 1.00 atm, (mg/L).	
$P$ =	atmospheric pressure, (atm).	
$P_{ww}$ =	partial pressure of water vapour, (atm),	
$P_1$ =	concentration of organic phosphorous, (mg-P/L)	
$P_2^*$ =	the local concentration of dissolved phosphorous, (mg-P/L).	
$P_N^*$ =	preference factor for ammonia nitrogen,	( 0.0 - 1.0 )
$r$ =	roughness height , (metres).	
$R_j$ =	concentration of the non-conservative constituent j, (mg/L).	
$S_{gs}$ =	Specific gravity of sand	
$S$ =	Source term, (gm/s/m <sup>2</sup> /meter of depth)	
$C_{eq}$ =	Equilibrium concentration (transport potential), (mg/L)	
$C$ =	Sand concentration in the water	
$t_c$ =	Characteristic time for effecting the transition, this depends on whether deposition or erosion is occurring, (s).	
$T$ =	temperature in ° C.	
$T_k$ =	water temperature in ° K , within the range 0.0 to 40.0 ° C.	
$u_m$ =	mean flow velocity, (m/sec).	
$u_s$ =	water velocity at the surface, (m/sec).	
$u_*$ =	shear velocity, (m/sec).	
$u^*$ =	bed shear velocity in English units for Ackers White formulation, (ft/s)	
$v$ =	water velocity near the bed, (m/s).	
$v^*$ =	water velocity in English units for Ackers White formulation, (ft/s)	
$V_b$ =	effective bed settling velocity, (m/sec)	
$V_k$ =	Von Karman's Constant	
$V_s$ =	particle settling velocity, (m/sec).	
$X_t$ =	the value of the coefficient at the local computed temperature	
$X_{20}$ =	the value of the coefficient at 20°C	
$z$ =	depth variable,(m).	
$z_d$ =	depth below water surface for 3-D (m)	

