4. GOVERNING EQUATIONS FOR HEAT BUDGET COMPUTATION

4.1 Introduction

In RMA-11 the dependent variable modelled when simulating heat transport is temperature T. The truly consistent parameter should be concentration of stored heat C_H which has units kJ/m^3. The heat capacity of water c and density may be used to relate these parameters. That is:

$$C_H = c\rho T$$

where

C_H	=	Heat content of water, (kJ/m^3)
c	=	Specific heat of water, (kJ/kg/Deg C)
ρ	=	Density of water, (kg/m^3).
T	=	Temperature, (Deg C).

The source rate term for heat at the water surface G_T may be written as

$$G_T = \frac{H_N}{3600\ c\ \rho}$$

where

G_T	=	Boundary temperature source rate (m Deg C/s)
H_N	=	Net energy flux passing the air water interface, ($kJ/m^2/hr$).

The approach used in RMA-11 consistent with QUAL2E and other literature is to assume that heat is transferred from various energy sources. So that:

$$H_N = H_{SN} + H_{AN} - (H_B + H_E + H_C)$$

where

H_{SN}	=	Net short-wave influx, ($kJ/m^2/hr$)
H_{AN}	=	Net long-wave influx, ($kJ/m^2/hr$)
H_B	=	Long-wave back radiation, ($kJ/m^2/hr$)
H_E	=	Conductive flux, ($kJ/m^2/hr$)
H_C	=	Evaporative flux, ($kJ/m^2/hr$)

The sections that follow will describe how each of these components is represented.

4.2 Net short-wave influx H_{SN}

The incoming short-wave radiation is that which passes directly from the sun to the earth's surface. The magnitude is a function of the altitude of the sun, the dampening effect of scattering and absorption in the atmosphere due to cloud cover, and reflection from the water surface. This functional form may be expressed by:

$$H_{SN} = H_0.\ a_\tau\ (1 - R_S).(1.-0.65.C_L^2)$$

where

H_0	=	Incoming solar short-wave to the earth's atmosphere, ($kJ/m^2/hr$)
a_τ	=	Atmospheric transmissivity
R_S	=	Albedo or reflection coefficient
C_L	=	Cloudiness, (expressed as a fraction 0 -1)

4 2.1 Incoming Solar Radiation (H_0)

The total incoming solar radiation over a time period is derived by integration of the solar constant accounting for latitude, the radius of the earth's orbit, the declination of the sun and the hour angles at the beginning and end of the time period. This relationship reported in Water Resources Engineers (1967) is

$$H_0 = \Gamma . \frac{H_{SC}}{r^2} [\sin \{\frac{\pi\phi}{180}.\}.\sin \{\delta(t_e-t_b)\}$$
$$+ \frac{12}{\pi} \cos\{\frac{\pi\phi}{180}\}.\cos\{ \delta\}(\sin\{\frac{\pi t_e}{12}\} - \sin\{\frac{\pi t_b}{12}\})]$$

where:

Γ = A correction factor for diurnal exposure to radiation flux
H_{SC} = Solar Constant, (4870.8 kJ/m^2/hr)
r = Normalised radius of earth orbit
ϕ = Latitude of site (degrees)
δ = Declination of the sun (degrees)
t_e = hour angle corresponding to end of time interval
t_b = hour angle corresponding to beginning of time interval

4.2.2 Normalised radius of earth orbit (r) and declination of the sun (δ)

The radius of the earth's orbit and the declination of the sun are dependent on the Julian day of the year and may be expressed as:

$$r = 1.0 + 0.017 . \cos\{ \frac{2\pi}{365}*(186- D_y)\}$$

and

$$\delta = 23.45. \frac{\pi}{180}.\cos\{(173 - D_y)\}$$

where

D_y = Julian day of the year.

4.2.3 Hour angles (t_e, t_b)

The hour angles associated with the beginning and end of a time step are functions of standard time, a correction for local time and for solar time
Thus

$$t_e = St_e - \Delta t_s + E_T - 12$$
$$t_b = St_b - \Delta t_s + E_T - 12$$

where

St_e = Standard time at the end of the interval (hours)
St_b = Standard time at the beginning of the interval (hours)
Δt_s = Difference between standard and local civil time (hours).
E_T = Time from a solar ephemeris, i. e., the difference in hours between "true solar time" and that computed on the basis of a yearly average.

In the above

$$E_T = 0.000121 - 012319 \sin\{\frac{2\pi}{365}(D_y - 1) - 0.0714\}$$

$$\Delta t_s = \frac{\varepsilon}{15}(L_{sm} - L_{lm})$$

where

ε = -1 for west longitude
+1 for east longitude

L_{sm} = Longitude of standard meridian (degrees)

L_{lm} = Longitude of local meridian (degrees)

4.2.4 Correction Factor for Diurnal exposure (Γ)

A correction factor must be applied to separate the period when the sun has risen from that when it is below the horizon. Thus:

$$\Gamma = 1$$

when $St_r < St_b$

or $St_e < St_s$

and

$$\Gamma = 0$$

when $St_s < St_b$

or $St_e < St_r$

where

St_r = Standard time of sunrise (hours).

St_s = Standard time of sunset (hours).

4.2.5 Sunrise and sunset (St_r, St_s)

Sunrise and sunset depend on the local latitude and declination of the sun and are given by:

$$St_r = 12 - \frac{12}{\pi} . \arccos\{ \tan(\frac{\pi\phi}{180}) \tan(\delta) \} + \Delta t_s$$

and

$$St_s = 24 - St_r + 2 . \Delta t_s$$

4.2.6 Atmospheric transmissivity (a_τ)

The atmospheric transmissivity is described in Water Resources Engineers (1967) in a series of parameters associated with mean atmospheric transmissivity and absorption. It is given by

$$a_\tau = \frac{\{a'' + 0.5.(1. - a' - d)\}}{\{1.0 - 0.5.R_s.(1.0 - a' + d)\}}$$

where

a' and a" = mean atmospheric transmission coefficients after scattering and adsorption

d = a dust attenuation coefficient that varies in the range (0 - 0.13).

a' and a" are given by expressions describing the clarity of the atmosphere and take the form:

$$a' = \exp\{ - [0.465 + 0.0408.P_{wc}] [0.129 + 0.171 \exp(-0.880.\theta_{am})]. \theta_{am}\}$$

and

$$a'' = \exp\{-[0.465 + 0.0408.P_{wc}]\,[0.179 + 0.421\exp(-0.721.\theta_{am})].\,\theta_{am}\}$$

in which

P_{wc} = The mean daily precipitable water content in the atmosphere, given by:

$$P_{wc} = 0.02936\exp(0.08802\,T_d)$$

where T_d = Dew point , (Deg C) given by

$$T_d = \ln\left[\frac{e_a + 2.8345}{8.8534}\right] / 0.054$$

e_a is the water vapour pressure in millibars (to be defined later),

and

θ_{am} = Optical air mass.

$$\theta_{am} = \frac{\exp\{-z / 771.46\}}{\{\sin\alpha + 0.15.(\frac{180\alpha}{\pi} + 3.885)^{-1.253}\}}$$

where

z = Elevation of the site, (m)

and α = Sun's altitude (radians) given by:

$$\alpha = \arcsin\{\sin(\frac{\pi\phi}{180})\sin\delta + \cos(\frac{\pi\phi}{180})\cos\delta\cos(\frac{\pi\tau}{12})\}$$

where t is the hour angle defined in a similar fashion to the hour angles above.

4.2.7 Reflectivity (R_S)

The reflection coefficient is a function of solar altitude and cloudiness C_L. Anderson (1954) gives empirical coefficients for determining R_S in the equation:

$$R_S = A\,\alpha^B$$

Cloudiness C_L	0 Clear		0.1 - 0.5 Scattered		0.6 - 0.9 Broken		1.0 Overcast	
Coefficients	A 1.18	B -0.77	A 2.20	B -0.97	A 0.95	B -0.75	A 0.35	B -0.45

4.3 Long-wave Atmospheric Radiation H_{AN}

Long wave radiation is mostly dependent on air temperature, it is also dependent on the cloudiness. A small fraction is reflected. The amount reflected is usually taken to 0.3 of the incoming radiation. The net flux can be expressed following Swinbank(1963), as

$$H_{AN} = 9.37E\text{-}06\,\sigma\,T_R^6\,(1 + 0.17\,C_L^2)\,(1-R_L)$$

where

σ = Stefan Boltzman constant,
= 2.0412 E-07 kJ/m^2/hr/deg K
T_R = Air temperature , (deg K).
R_L = Reflectivity of water surface (set = 0.03)

4.4 Long-wave Back Radiation H_B

Back radiation is that heat lost by the water through the air water interface. Using black body theory the back radiation may be expressed as:

$$H_B = 0.97\sigma T_S^4$$

where

T_S = The water surface temperature, (deg K).

4.5 Evaporation H_E

Evaporation is also a significant source of heat loss from the water body to the atmosphere. The rate of evaporation is converted to heat lost through the latent heat of vaporisation. so that

$$H_E = \gamma L E$$

where

γ = Specific weight of water,(kg/m^3)
E = Evaporation rate, (m/hr).
L = Latent heat of vaporisation, (kJ/kg)
= $2400 - 0.9\,T$
T = Water surface temperature, (deg C)

The evaporation rate is usually expressed as a function of the difference between the saturation vapour pressure of the air and the water vapour pressure and as function of local wind speed. It takes the form:

$$E = (a + b\,W)(e_s - e_a)$$

where

a and b area constants
W = wind speed, (m/s)
e_s = saturation vapour pressure of the air at the temperature of the water surface, (millibars).
= $8.8534 \exp(0.054\,T) - 2.8345$
e_a = water vapour pressure in the air, (millibars).
= $e_{wb} - 0.0006606\,P_a\,(T_a - T_{wb})\left(1 + \dfrac{T_{wb}}{872.78}\right)$
T_a = dry bulb air temperature, (deg C)
T_{wb} = wet bulb air temperature, (deg C)
e_{wb} = saturation vapour pressure at the wet bulb temperature
= $8.8534 \exp(0.054\,T_{wb}) - 2.8345$

Suggested values for a and b are given by Roesner (1969) as

a = 6.2E-06 m/hr/mbar
and b = 5.5E-06 m/hr/mbar per m/s of wind speed.

4.6 Conduction H_C

Heat transferred between the water and the atmosphere due to temperature differences not related to water vapour exchange is called conduction. It is usually assumed to be related to the same variables as evaporation and can be derived using a proportionality constant, known as Bowen's ratio. Bowen's ratio is usually expressed as:

$$B = \frac{H_c}{H_e} = C_B \left(\frac{T - T_a}{e_s - e_a}\right) \left(\frac{P_a}{1013.25}\right)$$

where

C_B = a coefficient = 0.6096 (metric units)

then

$$H_C = 0.6096\ L\ (a + b\ W)\ \left(\frac{P_a}{1013.25}\right) (T - T_a)$$